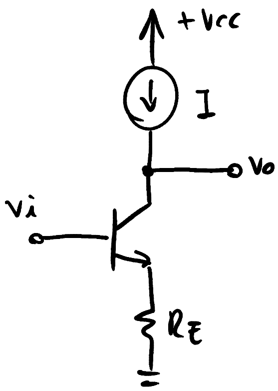
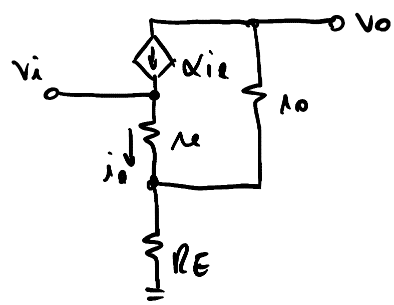
Nota de aula CEA → Análise CE - RE

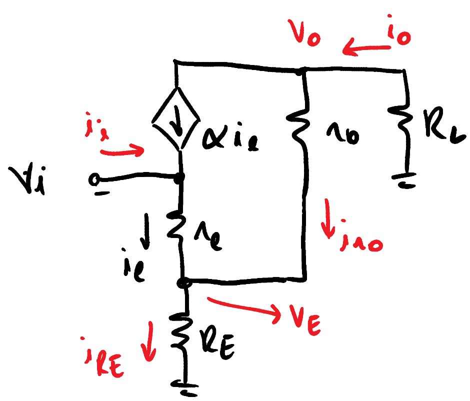


→ Assumindo uma fonte de corrente ideal o modelo de pequenos sinais se torna:



→ Cálculo Rin

↳ Para o cálculo de Rin iremos assumir uma carga na saída do amplificador e então verificar como esta irá afetar a entrada.



• Levantando a LKC no nó Vo

$$i_o = \alpha i_e + i_{ro}$$

$$-\frac{V_o}{R_L} = \alpha\left(\frac{V_i - V_E}{r_e}\right) + \frac{V_o - V_E}{r_o}$$

• Levantando a LKC no nó VE

$$i_e + i_{ro} = i_{RE}$$

$$\frac{V_i - V_E}{r_e} + \frac{V_o - V_E}{r_o} = \frac{V_E}{R_E}$$

→ $i_i = i_e(1-\alpha) = \dfrac{i_e}{\beta+1}$

→ Equacionando:

$$\begin{cases} -\dfrac{V_o}{R_L} = \dfrac{\alpha}{r_e}\cdot(V_i - V_E) + \dfrac{V_o - V_E}{r_o} \\ \dfrac{V_E}{R_E} = \dfrac{V_i - V_E}{r_e} + \dfrac{V_o - V_E}{r_o} \end{cases}$$

$$V_E\left(\frac{1}{R_E} + \frac{1}{r_e} + \frac{1}{r_o}\right) = \frac{V_i}{r_e} + \frac{V_o}{r_o}$$

$$V_E = (R_E /\!/ r_e /\!/ r_o)\left(\frac{V_i}{r_e} + \frac{V_o}{r_o}\right)$$

→ Assim:

$$-\frac{v_o}{R_L} = \frac{\alpha}{r_e} v_i + \frac{v_o}{r_o} - \left(\frac{\alpha}{r_e} + \frac{1}{r_o}\right) \cdot (R_E // r_e // r_o) \left(\frac{v_i}{r_e} + \frac{v_o}{r_o}\right)$$

$$-\frac{v_o}{R_L} = \frac{\alpha}{r_e} v_i + \frac{v_o}{r_o} - \frac{\alpha r_o + r_e}{\cancel{r_e r_o}} \cdot \frac{R_E \cancel{r_e r_o}}{R_E r_e + R_E r_o + r_e r_o} \left(\frac{v_i}{r_e} + \frac{v_o}{r_o}\right)$$

$$-\frac{v_o}{R_L} = \frac{v_i}{r_e}\left[\alpha - \frac{\alpha r_o R_E + R_E r_e}{R_E r_e + R_E r_o + r_e r_o}\right] + \frac{v_o}{r_o}\left[1 - \frac{\alpha r_o R_E + R_E r_e}{R_E r_e + R_E r_o + r_e r_o}\right]$$

$$-\frac{v_o}{R_L} = \frac{v_i}{r_e}\left[\frac{\alpha R_E r_e + \cancel{\alpha} R_E r_o + \alpha r_e r_o - \cancel{\alpha} r_o R_E - r_e R_E}{R_E r_e + R_E r_o + r_e r_o}\right] + \frac{v_o}{r_o}\left[\frac{\cancel{R_E r_e} + R_E r_o + r_e r_o - \alpha r_o R_E - \cancel{R_E r_e}}{R_E r_e + R_E r_o + r_e r_o}\right]$$

$$-\frac{v_o}{R_L} = \frac{v_i}{\cancel{r_e}}\left[\frac{\cancel{r_e}(\alpha R_E + \alpha r_o - R_E)}{R_E r_e + R_E r_o + r_e r_o}\right] + \frac{v_o}{\cancel{r_o}}\left[\frac{\cancel{r_o}(R_E + r_e - \alpha R_E)}{R_E r_e + R_E r_o + r_e r_o}\right]$$

$$v_i\left(\frac{\alpha r_o - (1-\alpha) R_E}{R_E r_e + R_E r_o + r_e r_o}\right) = -v_o\left[\frac{1}{R_L} + \frac{r_e + R_E(1-\alpha)}{R_E r_e + R_E r_o + r_e r_o}\right]$$

$$v_i\left(\frac{\alpha r_o - (1-\alpha) R_E}{R_E r_e + \cancel{R_E r_o} + r_e r_o}\right) = -v_o\left[\frac{R_E r_e + R_E r_o + r_e r_o + R_L r_e + R_E R_L(1-\alpha)}{R_L(R_E r_e + \cancel{R_E r_o} + r_e r_o)}\right]$$

$$\frac{v_o}{v_i} = \frac{-R_L(\alpha r_o - (1-\alpha)R_E)}{R_E r_e + R_E r_o + r_e r_o + R_L r_e + R_E R_L(1-\alpha)} \times \frac{\beta+1}{\beta+1}$$

$$\boxed{\frac{v_o}{v_i} = \frac{-R_L(\beta r_o - R_E)}{R_E r_\pi + (\beta+1)R_E r_o + r_o r_\pi + R_L r_\pi + R_E R_L}}$$

→ Com isso,

$$v_E = \frac{R_E r_e r_o}{R_E r_e + r_e r_o + R_E r_o}\left(\frac{v_i}{r_e} + \frac{v_o}{r_o}\right)$$

$$v_E = \frac{R_E r_o}{R_E r_e + R_E r_o + r_e r_o} \cdot v_i + \frac{R_E r_e v_i}{R_E r_e + R_E r_o + r_e r_o}\left[\frac{-R_L(\beta r_o - R_E)}{R_E r_\pi + (\beta+1) R_E r_o + r_o r_\pi + R_L r_\pi + R_E R_L}\right]$$

$$v_E = \frac{R_E \cdot v_i}{R_E r_e + R_E r_o + r_e r_o}\left[r_o - \frac{R_L r_e(\beta r_o - R_E)}{R_E r_\pi + (\beta+1)R_E r_o + r_o r_\pi + R_L r_\pi + R_E R_L}\right]$$

$$V_E = \frac{R_E V_i}{R_E r_e + R_E r_o + r_e r_o}\left[\frac{r_o R_E r_\pi + (\beta+1) R_E r_o^2 + r_o^2 r_\pi + R_L r_o r_\pi + R_E R_L r_o - R_L r_e \beta r_o + R_L R_E r_e}{R_E r_\pi + (\beta+1) R_E r_o + r_o r_\pi + R_L r_\pi + R_E R_L}\right]$$

• Manipulando a expressão

$$V_E = \frac{R_E \cdot V_i}{R_E r_e + \cancel{R_E r_o} + r_e r_o}\left[\frac{(r_o(\beta+1) + R_L)(R_E r_o + \cancel{R_E r_e} + r_e r_o)}{(R_E r_e + R_E r_o + r_o r_e)(\beta+1) + R_L(r_\pi + R_E)}\right]$$

$$\boxed{V_E = \frac{R_L R_E + r_o R_E(\beta+1)}{R_L(r_\pi + R_E) + (\beta+1)(R_E r_e + R_E r_o + r_o r_e)} \cdot V_i}$$

→ Finalmente

$$i_e = \frac{V_i - V_E}{r_e} = \frac{V_i}{r_e} - \frac{R_L R_E + r_o R_E(\beta+1) \cdot V_i}{r_e\left[R_L(r_\pi + R_E) + (\beta+1)(R_E r_e + R_E r_o + r_o r_e)\right]}$$

$$i_e = V_i \cdot \frac{R_L(r_\pi + \cancel{R_E}) + (\beta+1)(R_E r_e + \cancel{R_E r_o} + r_o r_e) - \cancel{R_L R_E} - r_o \cancel{R_E(\beta+1)}}{r_e\left[R_L(r_\pi + R_E) + (\beta+1)(R_E r_e + R_E r_o + r_o r_e)\right]}$$

$$i_e = V_i \cdot \frac{(\beta+1)(R_L \cancel{r_e} + R_E \cancel{r_e} + r_o \cancel{r_e})}{\cancel{r_e}\left[R_L(r_\pi + R_E) + (\beta+1)(R_E r_e + R_E r_o + r_o r_e)\right]}$$

$$i_i = \frac{i_e}{\beta+1} = V_i \cdot \frac{R_L + R_E + r_o}{R_L(r_\pi + R_E) + (\beta+1)(R_E r_e + R_E r_o + r_o r_e)}$$

$$\rightarrow \boxed{R_{in} = \frac{V_i}{i_i} = \frac{R_L(r_\pi + R_E) + (\beta+1)(R_E r_e + R_E r_o + r_o r_e)}{R_L + R_E + r_o}}$$

→ Assumindo o caso original, onde $R_L = \infty$

$$\boxed{R_{in}\Big|_{R_L=\infty} = r_\pi + R_E}$$

• obs: O que ocorre se R_L é finito:

$$R_L = 0 \rightarrow R_{in}\Big|_{R_L=0} = \frac{R_E r_e + R_E r_o + r_o r_e}{R_E + r_o}$$

$$r_o \gg R_L \rightarrow R_{in} \cong (\beta+1)(R_E + r_e)$$

→ Note que o CE-RE é um amplificador cuja Rin depende da carga, logo em uma análise multiestágio, deve-se conhecer toda a carga no coletor do transistor!

• Cálculo de Avo

→ A partir da análise anterior, sabemos que:

$$A_v = \frac{v_o}{v_i}\Big|_{R_L \neq \infty} = \frac{-R_L(\beta r_o - R_E)}{R_E r_\pi + (\beta+1)R_E r_o + r_o r_\pi + R_L r_\pi + R_E R_L}$$

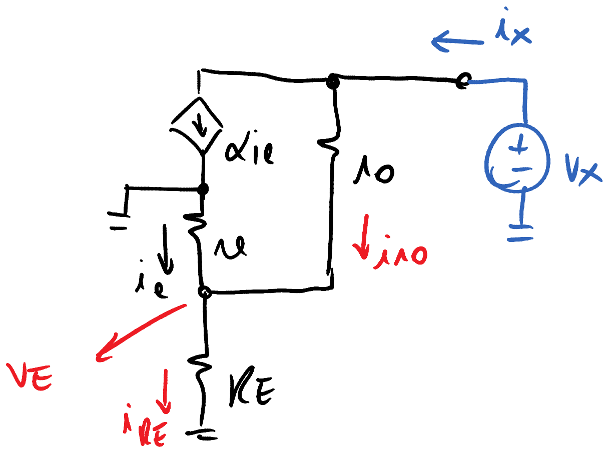
$$\rightarrow A_v = \frac{-R_L(\beta r_o - R_E)}{(\beta+1)(R_E r_e + R_E r_o + r_o r_e) + R_L(R_E + r_\pi)}$$

→ Assumindo agora $R_L = \infty$

$$\boxed{A_{vo} = \frac{-\beta r_o + R_E}{R_E + r_\pi}}$$

• Cálculo de Ro

→ Redesenhando



→ Analisando as LKC's

$$\begin{cases} i_x = \alpha i_e + i_{ro} \\ i_e + i_{ro} = i_{RE} \end{cases}$$

$$\begin{cases} i_x = \alpha\left(-\dfrac{V_E}{r_e}\right) + \dfrac{V_x - V_E}{r_o} \\ -\dfrac{V_E}{r_e} + \dfrac{V_x - V_E}{r_o} = \dfrac{V_E}{R_E} \end{cases}$$

$$\rightarrow \frac{V_x}{r_o} = V_E\left(\frac{1}{R_E} + \frac{1}{r_o} + \frac{1}{r_e}\right)$$

$$\boxed{V_E = \frac{R_E /\!/ r_o /\!/ r_e}{r_o} \cdot V_x}$$

→ Logo,

$$\frac{i_x}{v_x} = -\frac{\alpha}{r_e} \cdot \frac{R_E /\!/ r_o /\!/ r_e}{r_o} + \frac{1}{r_o} - \frac{R_E /\!/ r_o /\!/ r_e}{r_o^2}$$

$$\frac{i_x}{v_x} = \frac{-\alpha}{\cancel{r_e r_o}} \cdot \frac{R_E \cancel{r_e r_o}}{R_E r_e + R_E r_o + r_e r_o} + \frac{1}{r_o} - \frac{1}{r_o^{\cancel{2}}} \cdot \frac{R_E r_e \cancel{r_o}}{R_E r_e + R_E r_o + r_e r_o}$$

$$\frac{i_x}{v_x} = \frac{-\alpha R_E r_o + \cancel{R_E r_e} + R_E r_o + r_e r_o - \cancel{R_E r_e}}{r_o(R_E r_e + R_E r_o + r_e r_o)}$$

$$\frac{i_x}{v_x} = \frac{r_e \cancel{r_o} + (1-\alpha) R_E \cancel{r_o}}{\cancel{r_o}(R_E r_e + R_E r_o + r_e r_o)} \times \frac{(\beta+1)}{(\beta+1)}$$

$$\frac{i_x}{v_x} = \frac{r_\pi + R_E}{R_E r_\pi + r_\pi r_o + (\beta+1) R_E r_o}$$

$$R_o = \frac{v_x}{i_x} = \frac{R_E r_\pi + r_\pi r_o + (\beta+1) R_E r_o}{r_\pi + R_E}$$

$$R_o = \frac{R_E r_\pi}{r_\pi + R_E} + \frac{r_o \cancel{(r_\pi + R_E)}}{\cancel{r_\pi + R_E}} + \frac{\beta R_E r_o}{R_E + r_\pi}$$

$$R_o = R_E // r_\pi + r_o + \frac{\beta R_E}{R_E + r_\pi} \cdot r_o$$

$$\therefore \quad g_m = \frac{I_C}{V_T} = \frac{\alpha I_E}{V_T} = \frac{\alpha}{r_e} = \frac{\beta}{(\beta+1) r_e} = \frac{\beta}{r_\pi}$$

$$\rightarrow \beta = g_m r_\pi$$

• Assim $\rightarrow$ $$R_o = R_E // r_\pi + r_o + \frac{g_m r_\pi R_E}{R_E + r_\pi} \cdot r_o$$

$$\boxed{R_o = R_E // r_\pi + r_o + g_m (R_E // r_\pi) r_o}$$